

The plasma state: screening and oscillations

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Outline

1. Plasmas in the Universe and in the laboratory
2. Debye shielding and quasi-neutrality
3. Plasma oscillations

Reading

Boyd & Sanderson, ch. 1

Bittencourt, ch. 1

Fitzpatrick, ch. 1

Notes: chapter 1, §1.1–1.3

Problems: set 1

Plasmas in the Universe and in the laboratory

What is a plasma?

Key result: definition

A plasma is a **quasi-neutral** gas of charged (and possibly neutral) particles that exhibits **collective behaviour**.

Three conditions, each derived in this lecture:

$$\lambda_D \ll L$$

charges are screened: quasi-neutrality

$$N_D = \frac{4\pi}{3} n \lambda_D^3 \gg 1$$

many particles per Debye sphere: weak coupling

$$\omega_p \tau_n \gg 1$$

oscillations faster than collisions with neutrals

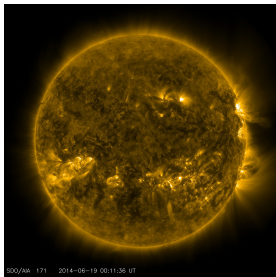
Historical note



Langmuir (1928) borrowed *plasma* from blood plasma for the ionised gas of discharge tubes, which carries electrons and ions. photo: Nobel Foundation

Plasmas are everywhere

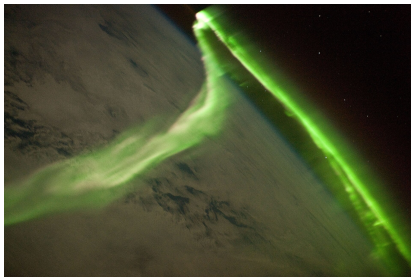
Almost all visible baryonic matter is ionised: stellar interiors and atmospheres, the interstellar and intracluster media, winds, magnetospheres, accretion discs.



Solar corona, 10^6 K
EUV, Fe IX 171 Å

NASA/SDO and the AIA science team

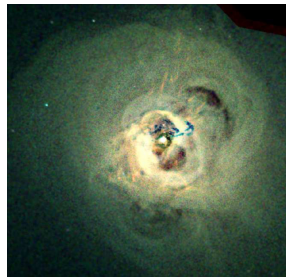
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Aurora: the ionosphere, 10^3 K
from the ISS

NASA / ISS Expedition 23

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Perseus cluster, 10^8 K
X-rays, bremsstrahlung

NASA/CXC/IoA/A. Fabian et al.

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Plasmas span ~ 30 orders of magnitude in density

	n_e [cm $^{-3}$]	T [K]	B [G]	
Intracluster medium	10^{-3}	$10^{7.5}$	10^{-6}	X-ray bremsstrahlung
Warm ionised ISM	10^{-1}	10^4	5×10^{-6}	pulsar DM and RM
Solar wind at 1 AU	5	10^5	5×10^{-5}	collisionless
Earth's ionosphere (F)	10^6	10^3	0.5	radio reflection
Solar corona	10^9	10^6	1–100	flares, reconnection
Solar photosphere	10^{13}	6×10^3	10^3 (spots)	weakly ionised
Tokamak core	10^{14}	10^8	5×10^4	magnetic confinement
Solar core	6×10^{25}	1.5×10^7	?	fully ionised
White-dwarf interior	10^{30}	10^7	up to 10^9	degenerate
Pulsar magnetosphere	$\sim 10^{11}$	–	10^{12}	$e^\pm$ pair plasma

Order-of-magnitude values. One theory must cover all of these $\Rightarrow$ dimensionless parameters: λ_D/L , N_D , Γ , degeneracy, v/ω_p , R_m .

Why not follow every particle?

N charges q_a , m_a , each obeying Newton's law with the Lorentz force,

$$m_a \frac{d^2 \mathbf{r}_a}{dt^2} = q_a \left[\mathbf{E}(\mathbf{r}_a, t) + \frac{1}{c} \dot{\mathbf{r}}_a \times \mathbf{B}(\mathbf{r}_a, t) \right], \quad a = 1, \dots, N,$$

with $\mathbf{E}$, $\mathbf{B}$ from Maxwell's equations sourced by the particles themselves,

$$\rho_c = \sum_a q_a \delta(\mathbf{r} - \mathbf{r}_a), \mathbf{j} = \sum_a q_a \dot{\mathbf{r}}_a \delta(\mathbf{r} - \mathbf{r}_a).$$

Impractical

$N \sim 10^{23}$ in a laboratory volume, $\sim 10^{57}$ in the Sun: $6N$ coupled ODEs with long-range forces.

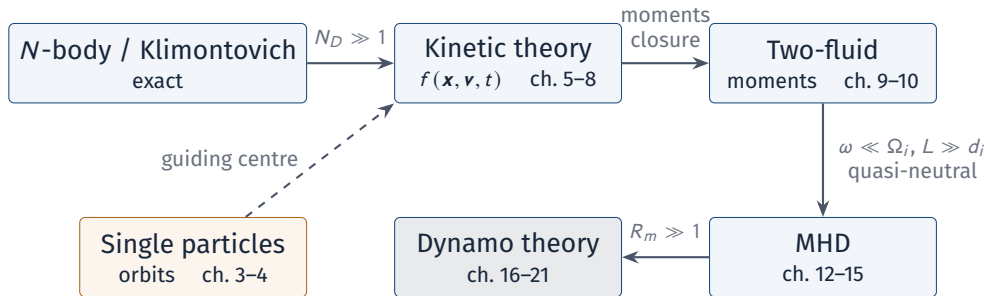
Not even desirable

we never know $6N$ initial conditions; walls, sources, collisions with neutrals and reactions are not in the equations.

Key result

We need a **statistical** description: the distribution function $f(\mathbf{x}, \mathbf{v}, t)$ (Vlasov, ch. 5) and its moments, the fluid variables n , $\mathbf{u}$, p .

Levels of description: the map of the course



Every arrow is an approximation controlled by a small parameter.

Today: the parameters that justify the first step, λ_D , ω_p and N_D , and the degree of ionisation.

Units: Gaussian throughout

Maxwell's equations

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 4\pi\rho_c, & \nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{B} &= \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

Lorentz force $q(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c)$

Recurring combinations

$B^2/8\pi$	magnetic pressure
$v_A = B/\sqrt{4\pi\rho}$	Alfvén speed
$\eta = c^2/4\pi\sigma$	magnetic diffusivity

SI form

SI $\rightarrow$ Gaussian:

$$\epsilon_0 \rightarrow \frac{1}{4\pi}, \quad \mu_0 \rightarrow \frac{4\pi}{c^2}, \quad \mathbf{B} \rightarrow \frac{\mathbf{B}}{c}$$

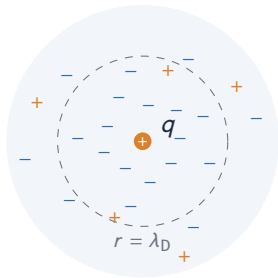
e.g. $B/\sqrt{\mu_0\rho} \rightarrow B/\sqrt{4\pi\rho}$. Converts equations, not numbers.

e	4.803×10^{-10} esu
1 G	10^{-4} T
1 eV	1.602×10^{-12} erg
	$\leftrightarrow 11\,604$ K

Temperatures in energy units, $k_B = 1$.

Debye shielding and quasi-neutrality

A test charge in a plasma



electrons gather around $q > 0$, ions are pushed away

Conductor: charges move until $\mathbf{E} = 0$ inside: perfect shielding.

Plasma: the charges are hot; thermal motion stops them piling up on q . Shielding is *partial* and acts over a finite distance.

Test charge q at the origin; species s with q_s , n_{s0} , T_s ; neutral, $\sum_s q_s n_{s0} = 0$. Poisson:

$$\nabla^2 \phi = -4\pi \left[q \delta(\mathbf{r}) + \sum_s q_s n_s(\mathbf{r}) \right]$$

We need $n_s(\mathbf{r})$ in the field of q .

The Boltzmann relation from force balance

Momentum equation of species s (a fluid of charges, ch. 9):

$$m_s n_s \frac{d\mathbf{u}_s}{dt} = q_s n_s \mathbf{E} - \nabla p_s$$

Assume: (1) static, inertia negligible; (2) electrostatic, $\mathbf{E} = -\nabla\phi$; (3) uniform T_s ; (4) ideal gas, $p_s = n_s T_s$.

$$0 = -q_s n_s \nabla\phi - T_s \nabla n_s \Rightarrow \frac{dn_s}{n_s} = -\frac{q_s}{T_s} d\phi \Rightarrow \ln n_s = -\frac{q_s \phi}{T_s} + C$$

With $n_s = n_{s0}$ far away, where $\phi = 0$:

Key result: Boltzmann relation

$$n_s(\mathbf{r}) = n_{s0} \exp\left(-\frac{q_s \phi(\mathbf{r})}{T_s}\right)$$

Linearisation

Poisson + Boltzmann: $\nabla^2 \phi = -4\pi q \delta(\mathbf{r}) - 4\pi \sum_s q_s n_{s0} e^{-q_s \phi / T_s}$, nonlinear in ϕ .
Far from the charge $|q_s \phi| \ll T_s$; expand $e^{-y} \simeq 1 - y$:

$$\sum_s q_s n_s \simeq \underbrace{\sum_s q_s n_{s0}}_{=0 \text{ (neutrality)}} - \phi \sum_s \frac{n_{s0} q_s^2}{T_s}$$

Substituting in Poisson gives the **screened Poisson equation**:

Key result

$$\nabla^2 \phi - \frac{\phi}{\lambda_D^2} = -4\pi q \delta(\mathbf{r}), \quad \frac{1}{\lambda_D^2} = \sum_s \frac{4\pi n_{s0} q_s^2}{T_s} \equiv \sum_s \frac{1}{\lambda_{Ds}^2}$$

The Debye length

Key result: electron Debye length

$$\lambda_{De} = \left(\frac{T_e}{4\pi n_e e^2} \right)^{1/2} = 743 \left(\frac{T_e/\text{eV}}{n_e/\text{cm}^{-3}} \right)^{1/2} \text{ cm}$$

SI: $\lambda_{De} = (\epsilon_0 T_e / n_e e^2)^{1/2}$

- ▶ hotter $\Rightarrow$ weaker screening: particles resist being pushed into the cloud
- ▶ denser $\Rightarrow$ stronger screening: more charges available
- ▶ species add in *parallel*: $\lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Di}^{-2}$; the best screener dominates
- ▶ ions screen only static or slow charges ($v \ll v_{ti}$); for faster phenomena $\lambda_D \rightarrow \lambda_{De}$

Solving the screened Poisson equation (1)

Spherical symmetry: write a Coulomb field times a correction, $\phi(r) = q f(r)/r$.

Product rule, $\nabla^2(gh) = g\nabla^2h + 2\nabla g \cdot \nabla h + h\nabla^2g$, with $g = 1/r$, $h = f$:

$$\nabla^2\left(\frac{f}{r}\right) = \frac{1}{r}\nabla^2f + 2\nabla\left(\frac{1}{r}\right) \cdot \nabla f + f\nabla^2\left(\frac{1}{r}\right)$$

Term by term, with $\nabla^2 = r^{-2}\partial_r(r^2\partial_r)$ and $\nabla^2(1/r) = -4\pi\delta(r)$:

$$\frac{1}{r}\nabla^2f = \frac{f''}{r} + \frac{2f'}{r^2}, \quad 2\nabla\left(\frac{1}{r}\right) \cdot \nabla f = -\frac{2f'}{r^2}, \quad f\nabla^2\left(\frac{1}{r}\right) = -4\pi f\delta(r)$$

The f' terms cancel, and $f\delta(r) = f(0)\delta(r)$:

Key result

$$\nabla^2\left(\frac{f}{r}\right) = \frac{f''}{r} - 4\pi f(0)\delta(r)$$

Solving the screened Poisson equation (2)

Insert $\phi = qf/r$ in $\nabla^2\phi - \phi/\lambda_D^2 = -4\pi q \delta(r)$:

$$\frac{qf''}{r} - 4\pi q f(0) \delta(r) - \frac{qf}{r\lambda_D^2} = -4\pi q \delta(r)$$

Delta functions match only if

For $r > 0$:

$f(0) = 1$ (bare Coulomb near q)

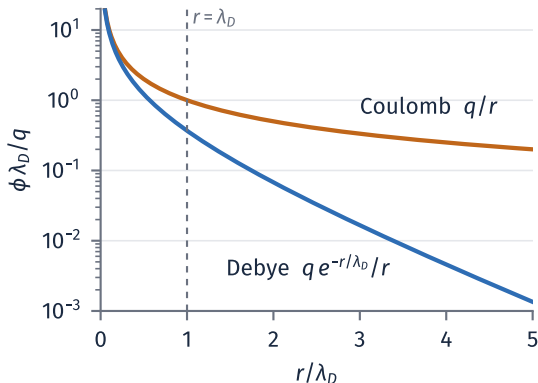
$$f'' = \frac{f}{\lambda_D^2} \Rightarrow f = A e^{-r/\lambda_D} + B e^{r/\lambda_D}$$

Bounded at infinity $\Rightarrow B = 0$; $f(0) = 1 \Rightarrow A = 1$:

Key result: Debye-Hückel potential

$$\phi(r) = \frac{q}{r} e^{-r/\lambda_D}$$

The Debye potential



Fourier view: transform

$\nabla^2 \phi - \phi / \lambda_D^2 = -4\pi q \delta$ with $\nabla^2 \rightarrow -k^2$:

$$\phi_k = \frac{4\pi q}{k^2 + \lambda_D^{-2}} = \frac{4\pi q}{k^2 \epsilon(k)}, \quad \epsilon(k) = 1 + \frac{1}{k^2 \lambda_D^2}$$

The plasma is a dielectric. It screens scales $k\lambda_D \lesssim 1$; at $k\lambda_D \gg 1$ the charge is bare. Inverting the transform gives back $q e^{-r/\lambda_D} / r$. [Notes: §1.2.3]

The screening cloud and its energy

Induced charge density (linearised): $\rho_{\text{ind}} = \sum_s q_s (n_s - n_{s0}) = -\phi / (4\pi\lambda_D^2)$. Its total:

$$\int \rho_{\text{ind}} d^3r = -\frac{1}{4\pi\lambda_D^2} \int_0^\infty \frac{q}{r} e^{-r/\lambda_D} 4\pi r^2 dr = -\frac{q}{\lambda_D^2} \underbrace{\int_0^\infty r e^{-r/\lambda_D} dr}_{\lambda_D^2} = -q$$

the cloud screens q *exactly*. Split $\phi = q/r + \phi_{\text{scr}}$; the cloud's potential at the charge is

$$\phi_{\text{scr}}(0) = \lim_{r \rightarrow 0} \frac{q}{r} \left(e^{-r/\lambda_D} - 1 \right) = \lim_{r \rightarrow 0} \frac{q}{r} \left(-\frac{r}{\lambda_D} + \dots \right) = -\frac{q}{\lambda_D}$$

so each particle interacts with its own cloud with energy $q\phi_{\text{scr}}(0) = -q^2/\lambda_D$.

Correlation energy and the plasma parameter

Every particle carries a cloud. Sum over particles, halve (each pair counted twice):

$$\frac{U_{\text{corr}}}{V} = \frac{1}{2} \sum_s n_{s0} q_s \phi_{\text{scr},s}(0) = -\frac{1}{2\lambda_D} \sum_s n_{s0} q_s^2$$

Hydrogen, $T_e = T_i = T$, $n_e = n_i = n$, so $\lambda_D^{-2} = 8\pi n e^2 / T$. Per particle:

$$w = \frac{U_{\text{corr}}}{2nV} = -\frac{e^2}{2\lambda_D}, \quad \frac{|w|}{T} = \frac{e^2}{2\lambda_D T} = \frac{1}{2\lambda_D} \cdot \frac{1}{8\pi n \lambda_D^2} = \frac{1}{16\pi n \lambda_D^3}$$

Key result

$$\frac{|w|}{T} = \frac{1}{12 N_D}, \quad N_D \equiv \frac{4\pi}{3} n \lambda_D^3$$

$|w| \ll T$ (ideal gas) $\Leftrightarrow$ many particles per Debye sphere.

Quasi-neutrality

Suppose thermal motions separate charges by $\delta n = n_i - n_e$ over a scale l :

$$\delta\phi \sim 4\pi e \delta n l^2$$

The separation stops once $e \delta\phi \sim T$:

Key result

$$\frac{\delta n}{n} \sim \frac{T}{4\pi n e^2 l^2} = \frac{\lambda_D^2}{l^2} \ll 1 \quad \text{for} \quad l \gg \lambda_D$$

- ▶ on scales $\gg \lambda_D$: $n_e = \sum_i Z_i n_i$ to high accuracy
- ▶ but $\mathbf{E} \neq 0$! Quasi-neutrality replaces Poisson's equation; $\mathbf{E}$ is then fixed by the electron force balance, not by $\nabla \cdot \mathbf{E} = 4\pi\rho_c \rightarrow$ ch. 9, generalised Ohm's law
- ▶ a gas of size $L \lesssim \lambda_D$ is a collection of charges, not a plasma

When does linearisation fail?

Linearisation needs $e\phi \ll T$, i.e. $r \gg b_0$ with

$$b_0 \equiv \frac{e^2}{T} \quad (\text{classical distance of closest approach}), \quad \frac{b_0}{\lambda_D} = \frac{1}{4\pi n \lambda_D^3} = \frac{1}{3N_D}$$

- ▶ the nonlinear core $r \lesssim b_0$ contains $\sim nb_0^3 \sim N_D^{-2} \ll 1$ particles
- ▶ treating charge density as continuous (mean field) needs many particles in the cloud: again $N_D \gg 1$
- ▶ the ratio λ_D/b_0 will reappear as the Coulomb logarithm $\ln \Lambda$, $\Lambda = 12\pi n \lambda_{De}^3 \rightarrow$ module 2a

Sheaths

Near a wall or probe, $e\phi \sim T$ and linearisation fails: a non-neutral *sheath* a few λ_D thick forms (Langmuir). Dust grains in the ISM charge up the same way.

Estimate: Debye lengths across the Universe

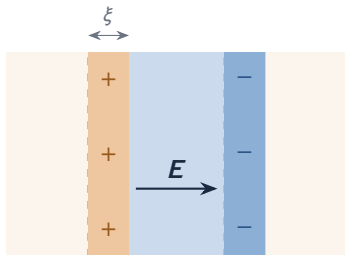
	n_e [cm ⁻³]	T	λ_{De}	N_D
Solar corona	10^9	10^6 K (86 eV)	0.2 cm	4×10^7
Warm ionised ISM	10^{-1}	10^4 K (0.86 eV)	20 m	4×10^9
Intracluster gas	10^{-3}	10^8 K (8.6 keV)	20 km	4×10^{16}
Tokamak	10^{14}	10^8 K	$70 \mu\text{m}$	10^8
Solar core	6×10^{25}	1.5×10^7 K	3×10^{-9} cm	≈ 10

Use $\lambda_{De} = 743 (T_{\text{eV}}/n)^{1/2}$ cm and $N_D = \frac{4\pi}{3} n \lambda_{De}^3$.

Most astrophysical plasmas are extremely ideal. The solar core is the exception: λ_{De} is comparable to the interparticle spacing $n^{-1/3} \approx 2.5 \times 10^{-9}$ cm, so the Debye-Hückel correction to the pressure is $\approx 1\%$, which helioseismology can detect.

Plasma oscillations

The electron sheet model



electrons displaced by ξ ; ions fixed

Surface charges $\pm en_0 \xi$ give (capacitor)
 $E = 4\pi en_0 \xi$.

Each electron obeys
 $m_e \ddot{\xi} = -eE = -4\pi n_0 e^2 \xi$

Key result: plasma frequency

$$\omega_{pe} = \left(\frac{4\pi n_e e^2}{m_e} \right)^{1/2}$$
$$= 5.64 \times 10^4 (n_e / \text{cm}^{-3})^{1/2} \text{ rad s}^{-1}$$

Independent of ξ and of wavelength (cold plasma).

Fluid derivation (1): model and equilibrium

Assumptions

- ▶ ions immobile, uniform density n_0 :
 $m_i \gg m_e$, so electrons respond much faster
- ▶ cold electrons, $T_e = 0$: no pressure
- ▶ electrostatic: $\mathbf{E} = -\nabla\phi$, $\mathbf{B} = 0$

Electron fluid + Poisson

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0$$

$$m_e n_e \left[\frac{\partial \mathbf{u}_e}{\partial t} + (\mathbf{u}_e \cdot \nabla) \mathbf{u}_e \right] = e n_e \nabla \phi$$

$$\nabla^2 \phi = -4\pi e (n_0 - n_e)$$

Method, used again for every wave and instability in the course:

1. find an **equilibrium**: here $\mathbf{u}_{e0} = 0$, $n_{e0} = n_0$, $\phi_0 = 0$
2. **perturb**: $n_e = n_0 + n_1$, $\mathbf{u}_e = \mathbf{u}_1$, $\phi = \phi_1$, with $n_1 \ll n_0$
3. **linearise**: keep terms of first order, drop products such as $n_1 \mathbf{u}_1$ and $(\mathbf{u}_1 \cdot \nabla) \mathbf{u}_1$

Fluid derivation (2): linearised equations

(1) Poisson: $\nabla^2 \phi_1 = 4\pi e n_1$

(2) continuity: $\frac{\partial n_1}{\partial t} = -n_0 \nabla \cdot \mathbf{u}_1$

(3) momentum: $m_e \frac{\partial \mathbf{u}_1}{\partial t} = e \nabla \phi_1$

Take ∂_t of (2) and the divergence of (3):

$$\frac{\partial^2 n_1}{\partial t^2} = -n_0 \nabla \cdot \frac{\partial \mathbf{u}_1}{\partial t} = -\frac{n_0 e}{m_e} \nabla^2 \phi_1$$

then use (1):

$$\frac{\partial^2 n_1}{\partial t^2} = -\frac{4\pi n_0 e^2}{m_e} n_1$$

Key result

$$\frac{\partial^2 n_1}{\partial t^2} + \omega_{pe}^2 n_1 = 0, \quad \omega_{pe}^2 = \frac{4\pi n_0 e^2}{m_e} \quad \Rightarrow \quad n_1 = A(\mathbf{r}) \cos[\omega_{pe} t + \alpha(\mathbf{r})]$$

An oscillation, not a wave

$$n_1(\mathbf{r}, t) = A(\mathbf{r}) \cos[\omega_{pe}t + \alpha(\mathbf{r})]$$

- ▶ same ω_{pe} as the sheet model, now for an arbitrary initial profile $A(\mathbf{r})$
- ▶ no spatial derivatives survive: each point oscillates on its own, up and down, with period $2\pi/\omega_{pe}$
- ▶ for a plane wave $e^{i(kx - \omega t)}$: $\omega = \omega_{pe}$ for every k , so the group velocity $d\omega/dk = 0$; no energy is carried away
- ▶ why? Cold plasma, $T_e = 0$: nothing couples neighbouring fluid elements. Pressure will do it (next slide)
- ▶ ions: same calculation with m_i , $\omega_{pi} = \omega_{pe}(Z^2 m_e/m_i)^{1/2} \approx \omega_{pe}/43$ for protons; with both species $\omega^2 = \omega_{pe}^2 + \omega_{pi}^2$

Thermal correction: the Bohm–Gross relation

Restore pressure: $m_e n_0 \partial_t \mathbf{u}_1 = e n_0 \nabla \phi_1 - \nabla p_1$, with adiabatic $p \propto n^\gamma$, so $p_1 = \gamma T_e n_1$.

Fast 1D compression (no heat flow over a wavelength), one degree of freedom: $\gamma = (f + 2)/f = 3$. Repeat the steps (divergence of momentum, ∂_t of continuity):

$$\frac{\partial^2 n_1}{\partial t^2} = -\omega_{pe}^2 n_1 + \frac{3T_e}{m_e} \nabla^2 n_1$$

Plane wave $n_1 \propto e^{i(kx - \omega t)}$: $\partial_t \rightarrow -i\omega$, $\nabla^2 \rightarrow -k^2$:

Key result: Bohm–Gross

$$\omega^2 = \omega_{pe}^2 + 3k^2 v_{te}^2, \quad v_{te} = (T_e/m_e)^{1/2}$$

Pressure couples neighbouring points: the oscillation now propagates, $v_g = 3k v_{te}^2 / \omega$. Kinetic theory adds Landau damping (ch. 6).

Time and length scales of the plasma response

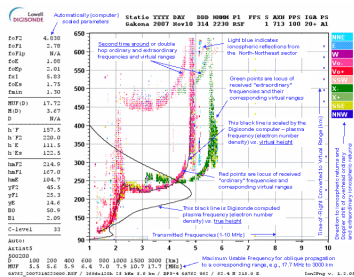
Key result

$$\omega_{pe} \lambda_{De} = v_{te}$$

Neutrality is restored in $\sim \omega_{pe}^{-1}$: the time for a thermal electron to cross λ_D .

- ▶ phenomena with $\omega \ll \omega_{pe}$ and $L \gg \lambda_D$: electrons keep the plasma neutral the MHD regime
- ▶ collisions with neutrals must not interrupt the oscillation: $\omega_{pe} \tau_n \gg 1$
- ▶ EM waves with $\omega < \omega_{pe}$ cannot propagate: they are reflected → ch. 11

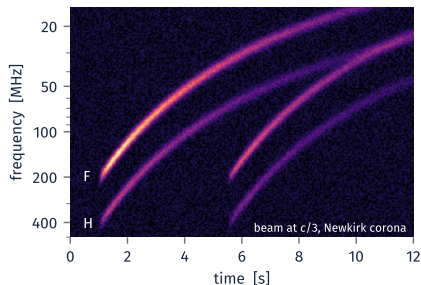
The plasma frequency in nature



HAARP / Wikimedia Commons

Ionosphere. Echoes come from where $f = f_{pe}(h)$. The highest, $f_oF_2 \approx 4.8$ MHz, gives the peak density $n_e = (f/8.98 \text{ kHz})^2 \approx 3 \times 10^5 \text{ cm}^{-3}$.

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synthetic: beam at $c/3$ in a Newkirk corona

Type III bursts. Flare beams radiate at f_{pe} (F) and $2f_{pe}$ (H); moving out into lower density, the frequency drifts down in seconds.

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Estimate: plasma frequencies

Practical form: $f_{pe} = \omega_{pe}/2\pi = 8.98 \text{ kHz } (n_e/\text{cm}^{-3})^{1/2}$

	$n_e [\text{cm}^{-3}]$	f_{pe}
Intracluster gas	10^{-3}	280 Hz
Solar wind at 1 AU	5	20 kHz
Ionosphere	10^6	9 MHz
Solar corona	10^9	280 MHz
Tokamak	10^{14}	90 GHz

Estimate: type III burst

A type III burst is observed at 100 MHz. What is the density of the emitting plasma?
 $n_e = (f/8.98 \text{ kHz})^2 \approx 1.2 \times 10^8 \text{ cm}^{-3}$: the low corona.

So far, and next

In this module

- ▶ charges are screened beyond $\lambda_D = (T/4\pi ne^2)^{1/2}$
- ▶ electrons oscillate at $\omega_{pe} = (4\pi ne^2/m_e)^{1/2}$;
 $\omega_{pe}\lambda_D = v_{te}$
- ▶ a plasma needs $L \gg \lambda_D$, $N_D \gg 1$, $\omega_{pe}\tau_n \gg 1$

Next: module 1b: coupling, degeneracy and the T - n diagram; ionisation and the Saha equation.